

Time Reversal Communication in Multi-Path Fading Channels with Pinholes

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The paper presents an analysis of the time reversal in multi-path Rayleigh-fading channels with N inputs (transmitters) and M outputs (receivers). The main issues addressed are the condition of statistical stability, the rate of information transfer and the effect of pinholes. The stability condition is proved to be $MC \ll N_{\text{eff}}B$ for broadband channels and $M \ll N_{\text{eff}}$ for narrowband channels where C is the symbol rate, B is the bandwidth and N_{eff} is the *effective* number of transmitters. It is shown that when the number of layers, $n - 1$, is relatively low compared to the logarithm of numbers of pinholes N_{eff} is given by n^{-1} times the harmonic mean of the number of transmitters and the numbers of pinholes at all layers. On the other hand, when the number of layers is relatively large the effective number of pinholes diminishes exponentially. The energy efficiency is shown to be optimal when the power supply is set to the noise level times BN_{eff} and that the maximal information rate is roughly BN_{eff} when the stability condition is violated.

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I. INTRODUCTION

Time reversal (TR) of waves has received great attention in recent years and been extensively studied for electromagnetic [2], [20], [32] as well as acoustic propagation (see [14] and the references therein). A striking effect of time reversal in randomly inhomogeneous media is the superresolution of refocal signals [1], [13] which implies low probability of intercept and holds high potential in technological applications such as communications [9], [25], [10], [17].

An issue prior to superresolution, however, is the condition of stability, namely how many antennas and how much bandwidth one needs to achieve statistical stability in TR so that the received signals are nearly deterministic, independent of the channel statistics? In this note I answer this question for multi-path Rayleigh fading channels, with multiple inputs and multiple outputs (MIMO), commonly used in wireless communication literature, see, e.g. [23] and I analyze the effect of multi-layer pinholes in Section V.

In the MIMO-TR communication scheme [9], [12], the M well-separated receivers first send a pilot signal to the N -element TRA which then uses the time-reversed version of the received signals to modulate the data symbols and retransmit them back to the receivers. One of the main results obtained here is that the time reversal process is statistically stable when

$$MC \ll N_{\text{eff}}B, \text{ for broadband channels} \quad (1)$$

$$M \ll N_{\text{eff}}, \text{ for narrowband channels} \quad (2)$$

where $C(\leq 2B)$ is the symbol rate, B is the bandwidth and N_{eff} is the *effective* number of transmitters. In the

presence of $(n - 1)$ -layer pinholes, I show that the effective number of transmitters is asymptotically equal to n^{-1} times the harmonic mean of the number of transmitters and the numbers of pinholes of all layers when these numbers are greater than 2^n . That is, the multi-layer pinholes have a screening effect on time-reversal transmission, reducing the effective number of the time-reversal elements by at least a factor of n . When the last condition is violated, the situation is even worse: N_{eff} diminishes exponentially fast as the number of layers increases, resulting in a tough environment to perform time reversal of good quality.

The LHS of (1) is the number of degrees of freedom per unit time in the constellation of intended data-streams while the RHS of (1) is roughly the number of degrees of freedom per unit time in the channel state information (CSI) received by TRA from the pilot signals. The latter has to be larger than the former in order to reverse the random scrambling by the channel and achieve deterministic outputs. The stability condition $N \gg 1$ for narrow-band channels or $B \gg \beta_c$ (the coherence bandwidth) for broadband channels, when M is small and the pinholes are absent, have been previously discussed in [1], [7], [8], [9], [20].

In Section V, I take into account the effect of noise and analyze the information rate in TR communication in noisy channels. I demonstrate a tradeoff between stability and spectral efficiency: the maximal information rate $R \sim BN_{\text{eff}}$ is achieved when the inequality in (1)-(2) is reversed. Also, for a given level of noise the energy efficiency is optimized when the power level is set to the noise level times $N_{\text{eff}}B$.

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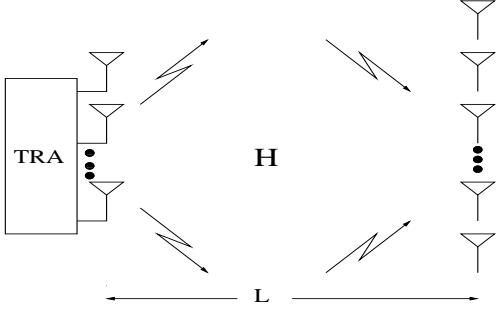


FIG. 1: MIMO Channel

II. TR-MIMO COMMUNICATION

First let us review the MIMO-TR communication scheme as described in [12].

The M receivers located at $\mathbf{y}_j, j = 1, \dots, M$ first send a pilot signal $\int e^{i\omega t} g(\omega) d\omega \delta(\mathbf{x}_j - \mathbf{y}_i)$ to the N -element TRA located at $\mathbf{x}_i, i = 1, \dots, N$ which then use the time-reversed version of the received signal $\int e^{i\omega t} g(\omega) H(\mathbf{y}_j, \mathbf{x}_i; \omega) d\omega$ to encode a stream of symbols and retransmit them back to the receivers. Here H is the transfer function of the propagation channel at the frequency ω from point \mathbf{y} to \mathbf{x} and $g^2(\omega)$ is the power density at ω . Let $\mathbf{H}(\omega) = [H_{ij}(\omega)]$ be the transfer matrix between the transmitters and receivers where $H_{ij}(\omega) = H(\mathbf{x}_i, \mathbf{y}_j; \omega)$. The reciprocity implies that $\mathbf{H}(\omega)$ is symmetric and the relation $\mathbf{H}^*(\omega) = \mathbf{H}(-\omega)$ where $*$ stands for complex conjugation. Let us assume that g is a smooth and rapidly decaying function with effective support of size B such as $g^2(\omega) = (2\pi)^{-1/2} \exp(-\frac{|\omega - \omega_0|^2}{2B^2})$. Naturally the relative bandwidth B/ω_0 is less than unity. We have chosen the time unit such that the speed of propagation is one.

Let us assume that the separation L between the TRA and the receivers is much larger than the spacing within the TRA-elements and the receivers. The signal vector $\mathbf{S} = (S_j)$ arriving at the receiver with delay $L + t$ is then given by [1], [6]

$$S_j(t) = \sum_{l=1}^W \sum_{i=1}^M m_i(\tau_l) \int e^{-i\omega(t-\tau_l)} g(\omega) \times \sum_{k=1}^N H_{jk}(\omega) H_{ik}^*(\omega) d\omega \quad (3)$$

where $m_j(\tau_l), l = 1, \dots, W \leq \infty$ is a stream of symbols intended for the j -th receiver transmitted at times $\tau_l = l\tau, \tau > 0$. In vector notation, we have $\mathbf{S} = \sum_{l=1}^W \int e^{-i\omega(t-\tau_l)} g(\omega) \mathbf{H} \mathbf{H}^\dagger(\omega) \mathbf{m}(\tau_l) d\omega$ where \mathbf{H}^\dagger is the conjugate transpose of \mathbf{H} and $\mathbf{m}(\tau_l) = (m_j(\tau_l))$.

Let us note that while all the TRA-elements are coordinated and synchronized the receivers do not have the knowledge about the channel and can not coordinate in decoding the total signals received. As a consequence, the co-channel interference from multiple users can be a serious problem [23]. An advantage of the time reversal scheme is the possibility to use the (statistical) stability property to achieve the following asymptotic

$$\int e^{-i\omega(t-\tau_l)} g(\omega) \sum_{k=1}^N H_{jk}(\omega) H_{ik}^*(\omega) d\omega \sim \delta_{ij} \delta(t - \tau_l)$$

so that $S_j(t) \sim \sum_{l=1}^W m_l(\tau_l) \delta(t - \tau_l)$ and each receiver receives the intended symbols without interference.

III. STATISTICAL STABILITY

One of the main goals of the present note is to characterize the stability regime for the important channel model of multi-path Rayleigh fading in which H_{ij} are i.i.d. $CN(0, 1)$, the zero-mean, variance-one circularly symmetric complex-Gaussian random variables. For simplicity, I assume that $|m_i(\tau_l)| = \mu, \forall i, l$. The multi-path Rayleigh fading after proper normalization is a simplified model for richly scattering environment when the spacing within the transmitters and receivers is larger than the coherence length ℓ_c of the channel. In general, the coherence length is inversely proportional to the angular spread [23] and sometimes can be computed explicitly in terms of physical properties of the channel [12]. For diffuse waves the coherence length is known to be on the scale of wavelength [28], [27].

Let us calculate the mean and the variance of the signals with respect to the ensemble of the channel. We use \mathbb{E} to denote the channel ensemble average. Consider the quantity $\mathbf{H} \mathbf{H}^\dagger \mathbf{m}$. By the Gaussian rule for the calculation of moments we have

$$\mathbb{E} \mathbf{H} \mathbf{H}^\dagger \mathbf{m} = N \mathbf{m} \quad (4)$$

$$\mathbb{E} |(\mathbf{H}(\omega) \mathbf{H}^\dagger(\omega) \mathbf{m})_j|^2 = N^2 |m_j|^2 + N \sum_{i=1}^M |m_i|^2 \quad (5)$$

so that $\mathbb{E} \mathbf{S} = N \mathbf{B} \mathbf{m} \sum_{l=1}^W e^{-i\omega_0(t-\tau_l)} e^{-B^2(t-\tau_l)^2}$. Let $\tau \geq (2B)^{-1}$ so that the summation in $\mathbb{E} \mathbf{S}$ is B -uniformly bounded as $W \rightarrow \infty$.

We measure the statistical stability of the signals by the normalized variance of the signals at the receivers

$$\mathcal{V}_j(\tau_n) = \frac{V_j(\tau_n)}{|\mathbb{E} S_j|^2(\tau_n)}, \quad V_j(\tau_n) \equiv \mathbb{E} |S_j|^2(\tau_n) - |\mathbb{E} S_j(\tau_n)|^2,$$

$\forall j, n$ and say that the signals are stable when $\mathcal{V}_j(\tau_n) \rightarrow 0, \forall j, n$.

Let β_c be the coherence bandwidth of the channel such that

$$\begin{aligned} \mathbb{E} \left\{ \left(\mathbf{H}(\omega) \mathbf{H}^\dagger(\omega) \mathbf{m} \right)_j \left(\mathbf{H}(\omega') \mathbf{H}^\dagger(\omega') \mathbf{m} \right)_j^* \right\} \\ \approx \begin{cases} N^2 |m_j|^2 + N \sum_{i=1}^M |m_i|^2, & |\omega - \omega'| \leq \beta_c/2, \\ 0, & |\omega - \omega'| \gg \beta_c. \end{cases} \end{aligned}$$

The coherence bandwidth β_c is inversely proportional to the delay spread and hence the delay-spread-bandwidth product (DSB) is roughly $B\beta_c^{-1}$ [11], [12], [23]. In the diffusion approximation β_c is given by the Thouless frequency $D_B L^{-2}$ where D_B is the Boltzmann diffusion constant, equal to the energy transport velocity times the transport mean free path, and L the distance of propagation [19], [29].

The *broadband, frequency-selective* (BBFS) channel is naturally defined as having a large DSB, i.e. $B\beta_c^{-1} \gg 1$. Since $B < \omega_0$, $\omega \in [\omega_0 - B/2, \omega_0 + B/2]$ and $-\omega$ are separated by more than β_c in a broadband frequency-selective channel. The same holds for a *narrow-band, frequency-non-selective* (NBFN) channel defined as $B \leq \beta_c \ll \omega_0$.

Consider the NBFN case first. We have

$$V_j(t) \approx NB^2 \sum_{i=1}^M |m_i|^2 \left| \sum_{l=1}^W e^{i\omega_0 \tau_l} e^{-B^2(t-\tau_l)^2/2} \right|^2$$

In view of (5) the stability condition $N \gg M$ for NBFN channels then follows easily. The main focus of the paper, however, is the BBFS channels for which we have instead

$$\begin{aligned} V_j(t) &\approx NB \sum_{i=1}^M |m_i|^2 \sum_l \frac{2 \sin \frac{\beta_c}{2}(t - \tau_l)}{t - \tau_l} \\ &\quad \times \sum_{l'=1}^W e^{-i\omega_0(l'-l)\tau} e^{-B^2(l-l')^2\tau^2/2}. \end{aligned} \quad (6)$$

Several observations are in order. First, due to $\tau \geq (2B)^{-1}$ the summation over l' in (6) is convergent as $W \rightarrow \infty$ uniformly in l and B . Second, the summation over l is also convergent as $W \rightarrow \infty$ with the effective number of terms $\sim B\beta_c^{-1}$. As a result, it suffices to consider the case $W = O(B\beta_c^{-1})$ or equivalently $|\tau_1 - \tau_W| = O(\beta_c^{-1})$ for which we have the estimate $V_j \sim NBC \sum_{i=1}^M |m_i|^2$ where C is the number of symbols per unit time in *each* data-stream. It then follows that $V_j \rightarrow 0$ if and only if $NB \gg MC$ for BBFS channels. The transition to the condition $N \gg M$ for NBFN channels takes place when $B \sim C$, i.e. $\tau \sim B^{-1}$.

Since NB is the number of degrees of freedom in the channel state information collected at the TRA per unit time and MC is the number of degrees of freedom in the ensemble of messages per unit time the stability condition $NB \gg MC$ can be interpreted

as saying that in order to recover the deterministic messages, independent of the channel ensemble, and thus reverse the random scrambling by the channel the former must be much larger than the latter.

A detailed, rigorous analysis of the MIMO-TR channel modeled by a stochastic Schrödinger equation, in the parabolic approximation, with a random potential is given in [12].

IV. RATE OF INFORMATION TRANSFER

In this section we discuss the information rate for a memoryless channel based on the multi-path Rayleigh-fading transfer matrix defined above [31], [15]. In a memoryless channel an independent realization of the random transfer matrix is drawn after each delay spread. Let us assume as usual that the noise at the each receiver is additive-white-Gaussian-noise (AWGN) and that the input vector is multivariate Gaussian and that the channel, the receiver noise and the input are independent of one another.

According to Shannon's theorem [5] the information (in nats) that a symbol can convey on average is $2^{-1} \ln(1 + \text{SINR})$ where SINR, the signal-to-interference-noise ratio at each receiver, is given by the harmonic mean of the SIR, the signal-to-interference ratio and SNR, the signal-to-noise ratio, i.e. $\text{SINR} = (\text{SIR}^{-1} + \text{SNR}^{-1})^{-1}$. This formulation assumes that the interference is approximately AWGN, like the noise at the receivers. For the multi-path Rayleigh fading channel considered here the interference statistics is strictly speaking high-degree χ -square for which Gaussian statistics is a reasonable approximation for our purpose of deriving a rough estimate of the trade-off between the stability and information rate.

According to the preceding analysis $\text{SIR} \sim NB/(MC)$ in the case of BBFS channels and $\text{SIR} \sim N/M$ in the case of NBFN channels, independent of the power constraint. Let us set the covariance matrix of the receiver noises to be $\nu \mathbf{I}_M$, the M -dim identity. Suppose the average transmission power is constrained to P is constrained and all the transmit and receive antennas are identical. We have $\text{SNR} = P/(\nu MC)$. In the case of BBFS channels, the SINR is roughly

$$\text{SINR} \sim \left(\frac{MC}{NB} + \frac{\nu MC}{P} \right)^{-1} \quad (7)$$

and, in the case of NBFN channels, it is

$$\text{SINR} \sim \left(\frac{M}{N} + \frac{\nu MC}{P} \right)^{-1} \quad (8)$$

The optimal SINR is obtained by setting $N \sim P(\nu \max(B, C))^{-1}$ which is roughly $P/(vB)$ since

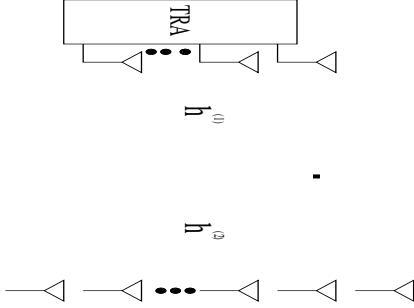


FIG. 2: Single-layer pinholes

$C \leq 2B$. The information rate R , given roughly by $2^{-1}MC \ln(1 + \text{SINR})$, achieves the maximum roughly equal to P/ν (in nat) at $MC \gg NB$ (for BBFS) or $N \gg M$ (for NBFN). That is, the channel capacity is linearly proportional to the power and is achieved at the expense of statistical stability of signals.

Consider the thermal noise power $\nu = k_B T B$ where k_B is the Boltzmann's constant and T the temperature. Then the above result implies that the energy cost per nat is $P/R \sim k_B T$ (with a constant close to unity) which is an extension to the TR-MIMO Rayleigh fading channel of the classical result derived for a simple SISO channel [24] (see also [21]).

V. PINHOLE EFFECT

Pinholes are degenerate channels that can occur in a wide family of channels, outdoor as well as indoor, see Figs. 2 and 3. While preserving the co-channel decorrelation, pinholes have been shown to severely limit the degrees of freedom and reduce the channel capacity [3], [16], [4]. In this section, I present an analysis of the pinhole effect on TR in Rayleigh fading channels to demonstrate similar effects on stability and information rate.

First, let us consider the simplest case of single-layer pinholes as illustrated in Fig. 2. Let $\mathbf{h}^{(1)}(\omega)$ be the $N \times K$ transfer matrix from the TRA to the pinholes and $\mathbf{h}^{(2)}(\omega)$ the $K \times M$ transfer matrix from the pinhole to the M receivers at frequency ω . The combined channel can be described by $\mathbf{H}(\omega) = \mathbf{h}^{(2)}(\omega)\mathbf{h}^{(1)}(\omega) = [h_{ik}^{(2)}(\omega)h_{kj}^{(1)}(\omega)]$ in which $h_{kj}^{(1)}$ and $h_{ij}^{(2)}$ are assumed to be i.i.d. $CN(0, \sigma_1)$ and $CN(0, \sigma_2)$, respectively.

At frequency ω , the mean signals received are given by $\mathbf{E}\mathbf{H}\mathbf{H}^\dagger \mathbf{m} = NK\sigma_1\sigma_2\mathbf{m}$ and the variance of the

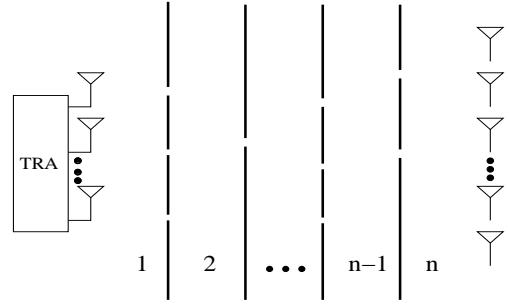


FIG. 3: Multi-layer pinholes

signal at frequency ω received at receiver i is given by

$$\begin{aligned} & \sum_{k=1}^K \mathbb{E}|h_{ik}^{(2)}|^2 \sum_{n=1}^M \mathbb{E}|h_{nk}^{(2)}|^2 |m_n|^2 \left(\sum_{j=1}^N \mathbb{E}|h_{kj}^{(1)}|^2 \right)^2 \\ & + \sum_{k=1}^K \mathbb{E}|h_{ik}^{(2)}|^2 \sum_{l=1}^K \sum_{n=1}^M \mathbb{E}|h_{nl}^{(2)}|^2 |m_n|^2 \sum_{j=1}^N \mathbb{E}|h_{kj}^{(1)}|^2 \mathbb{E}|h_{lj}^{(1)}|^2 \\ & + \sum_{k=1}^K (\mathbb{E}|h_{ik}^{(2)}|^2)^2 |m_i|^2 \left(\sum_{j=1}^N \mathbb{E}|h_{kj}^{(1)}|^2 \mathbb{E}|h_{kj}^{(1)}|^2 \right) \\ & \approx KN(MN + MK + 1)\sigma_1^2\sigma_2^2|\mu|^2. \end{aligned}$$

Taking into account the temporal aspect of the signal as before we obtain the normalized variance of the signal to the leading order ($N, K \gg 1$) for BBFS channels

$$\mathcal{V}_j \approx MCB^{-1}(N^{-1} + K^{-1}), \quad \forall j$$

and for NBFN channels

$$\mathcal{V}_j \approx M(N^{-1} + K^{-1}), \quad \forall j.$$

The result indicates that there is an *effective* number of TRA-elements given by $N_{\text{eff}} = NK(N + K)^{-1}$, namely one half of the harmonic mean of N and K , so that $\mathcal{V}_j \approx MCB^{-1}N_{\text{eff}}^{-1}$ for BBFS channels and $\mathcal{V}_j \approx MN_{\text{eff}}^{-1}$ for NBFN channels. The confirms the intuition that pinholes are choke-points that reduce the effective number of TRA-elements.

The previous case *without* pinholes corresponds to the limiting case $K \rightarrow \infty$. For a fixed K , however, the previous benefit of having large number of TRA elements ($N \gg 1$) disappears. The multiple antennas in TRA are essentially screened out by the pinholes and the effective number of TRA-elements becomes K .

The same analysis can be applied to channels with $(n - 1)$ layers of pinholes such as illustrated in Fig. 3. Let $K_k, k = 1, \dots, n - 1$ be the number of k -th layer

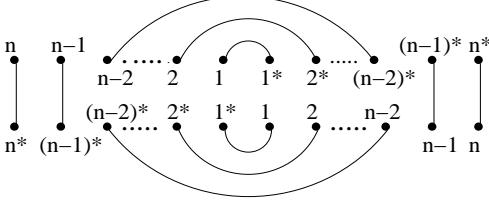


FIG. 4: Example of a simple graph

pinholes. Let $\mathbf{h}^{(k)}$ be the transfer matrix for the k -th stage channel whose entries are i.i.d. zero-mean, variance- σ_k Gaussian r.v.s. and let the transfer matrices of different stages be independent of one another.

I now show that to the leading order ($N, K_1, \dots, K_{n-1} \gg 2^n$) the normalized variance of the signal is given by $\mathcal{V} \approx MCB^{-1}N_{\text{eff}}^{-1}$ where the effective number of TRA-element N_{eff} is given by

$$N_{\text{eff}} = \left(N^{-1} + N_p^{-1} \right)^{-1}, \quad N_p = \left(\sum_{j=1}^{n-1} K_j^{-1} \right)^{-1};$$

namely n^{-1} times the harmonic mean of N, K_1, \dots, K_{n-1} . The effective number of pinholes N_p is always larger than N_{eff} and this again confirms the limiting nature of the pinholes. I sketch the proof below.

The calculation of the mean is straightforward: $\mathbb{E}\mathbf{H}\mathbf{H}^\dagger\mathbf{m} = NK_1 \dots K_{n-1}$. Let us analyze the second moment of entry a

$$\begin{aligned} & \mathbb{E}(\mathbf{H}\mathbf{H}^\dagger\mathbf{m})_a (\mathbf{H}\mathbf{H}^\dagger\mathbf{m})_a^* \\ &= \mathbb{E} \sum_{\substack{i_1, \dots, i_n \\ j_2, \dots, j_{n+1}}} h_{ai_n}^{(n)} h_{i_n, i_{n-1}}^{(n-1)} \dots h_{i_3, i_2}^{(2)} h_{i_2, i_1}^{(1)} h_{j_2, i_1}^{(1)*} h_{j_3, j_2}^{(2)*} \\ & \quad \times \dots h_{j_n, j_{n-1}}^{(n-1)*} h_{j_{n+1}, j_n}^{(n)*} m_{j_{n+1}} \sum_{\substack{i'_1, \dots, i'_n \\ j'_2, \dots, j'_{n+1}}} h_{ai'_n}^{(n)*} h_{i'_n, i'_{n-1}}^{(n-1)*} \\ & \quad \times \dots h_{i'_3, i'_2}^{(2)*} h_{i'_2, i'_1}^{(1)*} h_{j'_2, i'_1}^{(1)} h_{j'_3, j'_2}^{(2)} \dots h_{j'_n, j'_{n-1}}^{(n-1)} h_{j'_{n+1}, j_n}^{(n)} m_{j'_{n+1}}^*. \end{aligned}$$

I claim that according to the Gaussian rule the leading order terms of the second moment correspond to the *simple* graphs in which the arcs, connecting (un)primed indices to (un)primed indices, are nested and are bound by the ladder edges, connecting unprimed indices to primed indices. This includes the graph corresponding to $|\mathbb{E}\mathbf{H}\mathbf{H}^\dagger\mathbf{m}|^2$ which has no ladder edges. A simple graph is illustrated in Fig. 4.

The observation is proved by induction. When a new layer of pinholes, described by $\mathbf{h}^{(n+1)}$, is added, the number of graphs is doubled: one half of them contain the ladder edges connecting $h_{ai_{n+1}}^{(n+1)}$ to $h_{ai'_{n+1}}^{(n+1)*}$ and

$h_{j_{n+2}, j_{n+1}}^{(n+1)*}$ to $h_{j'_{n+2}, j'_{n+1}}^{(n+1)}$ while the other half contain the arcs connecting $h_{ai_{n+1}}^{(n+1)}$ to $h_{j_{n+2}, j_{n+1}}^{(n+1)*}$ and $h_{ai'_{n+1}}^{(n+1)*}$ to $h_{j'_{n+2}, j'_{n+1}}$. A moment of reflection reveals that the new pair of arcs impose *one* more constraint on the ranges of the indices than the new pair of ladder edges and yield a linear factor of K_{n+1} or M to the n -th order graphs while the new pair of ladder edges yield a quadratic factor MK_{n+1} to the graph corresponding to $|\mathbb{E}\mathbf{H}\mathbf{H}^\dagger\mathbf{m}|^2$ and K_{n+1}^2 to the rest of the n -th order simple graphs. Note that the structure of simple graphs is not changed by the new ladder edges and the total number of n -th order simple graphs is exactly $n+1$.

Collecting the terms corresponding to the simple graphs we have

$$\begin{aligned} & \mu^2 NM \prod_{i=1}^n \sigma_i^2 \prod_{j=1}^{n-1} K_j \\ & \times \left(\prod_{k=1}^{n-1} K_k + N \sum_{i=1}^{n-1} K_1 \dots \widehat{K}_i \dots K_{n-1} \right) \end{aligned}$$

where \widehat{K}_i means that K_i is absent in the product. Dividing it by $N^2 \prod_{i=1}^{n-1} K_i^2$ and accounting for the temporal aspect of transmission we obtain the claimed result.

To calculate the SINR we can substitute N_{eff} for N in (7). Even with unlimited power supply one would do best by choosing the optimal power level $P \sim B\nu N_{\text{eff}}$; higher power level is wasteful. The maximal information rate is roughly BN_{eff} which is still bounded by BN_p .

VI. CONCLUSIONS

I have analyzed the time reversal of propagation in multi-path Rayleigh-fading MIMO-channels with or without pinholes. The focus of the analysis is the stability condition, the multiplexing gain and their trade-off. The main results can be summarized as (i) that the stability holds when $MC \ll N_{\text{eff}}B$ for BBFS channels and $M \ll N_{\text{eff}}$ for NBFN channels where N_{eff} is the effective number of TRA-elements and (ii) that the maximal information rate BN_{eff} is attained when the power supply is set to the noise level times BN_{eff} and when the stability condition is reversed. They are a significant extension of the previously discussed conditions for stability [1], [7], [8], [9].

I have computed the *effective* number of TRA-element N_{eff} under the condition that the numbers of TRA-elements and the pinholes of each layer are much greater than 2^n , with $n-1$ being the number

of layers. In this extreme case the effective number of TRA-elements is asymptotically equal to n^{-1} times the harmonic mean of TRA-elements and the numbers of pinholes at all $n-1$ layers. In the opposite case the graph analysis shows that the normalized variance of signal grows exponentially with the number of pin-hole layers and consequently the rate of information transmission diminishes exponentially fast. In other words, a long chain of independently fluctuating media separated by a series of screens of sparse pinholes is detrimental to time reversal communication.

The estimate NB for the TR information rate in the absence of pinholes should be contrasted with the finding in [15], [31], [22], [30] that the capacity with CSI at

the receiver with M receive antennas (but not at the N transmit antennas) scales like $B \min(M, N) \ln \text{SNR}$ at high SNR. Note that their result does not include the interference due to non-cooperating multiuser receivers as is done here and is for narrow-band signals with $C = 2B$.

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